# The Structures of $\mathrm{BaCeO}_{3}, \mathrm{BaPrO}_{3}$ and $\mathrm{BaTbO}_{3}$ by Neutron Diffraction: Lattice Parameter Relations and Ionic Radii in $\boldsymbol{O}$-Perovskites 

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#### Abstract

The perovskites $\mathrm{BaCeO}_{3}, \mathrm{BaPrO}_{3}$ and $\mathrm{BaTbO}_{3}$ containing tetravalent rare earth ions have been investigated by powder neutron diffraction. All three show distortions from cubic symmetry. $\mathrm{BaTbO}_{3}$ is rhombohedral ( $R \overline{3} c$ ) and $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$ orthorhombic ( Pbnm ), but with $c / \sqrt{ } 2<a<b$. Atom positions have been determined from least-squares refinement of peak intensities, and $\mathrm{BaTbO}_{3}$ has been shown to become a $G$-type antiferromagnet below $36^{\circ} \mathrm{K}$. The rotations of octahedra necessary to produce the orthorhombic (Pbnm) structure are discussed and a relation between the lattice parameters and the metal oxygen distance has been derived. The influence of small distortions within the octahedra are considered, and the variability of ionic radii in the perovskite structure is discussed.


## Introduction

The class of mixed metal $\mathrm{ABO}_{3}$ oxides with perovskitelike structures is a large one, although few compounds have the ideal cubic unit cell (Goodenough \& Longo, 1970; Galasso, 1969). For tolerance factors $\dagger$ less than unity two types of distortion commonly occur, both of which involve rotations of the $\mathrm{BO}_{6}$ octahedra. The simple rhombohedral distortion is associated with a rotation about a trigonal axis (Fig. 1), lowering the symmetry to $R \overline{3} c$ (e.g. $\mathrm{LaAlO}_{3}$ ). A further reduction of symmetry to $R 3 c$ may be observed when the A and B atoms are displaced along a trigonal axis (e.g. $\mathrm{BiFeO}_{3}$ ). For larger deviations from the ideal ionic radius ratios the orthorhombic ( Pbnm ) $\mathrm{GdFeO}_{3}$ structure is observed (see Fig. 2).
The cooperative tilting of the octahedra in these compounds can produce considerable oxygen-atom displacements from the ideal perovskite positions although the distortion of the unit cell may only be small, and line splittings in X-ray powder photographs have often been overlooked. For structures containing heavy A and B atoms, extra reflexions arising principally from oxygen displacements may be difficult to detect in X ray work but are often observed with ease by neutron diffraction.
In this work we have determined the structures of $\mathrm{BaCeO}_{3}, \mathrm{BaPrO}_{3}$ and $\mathrm{BaTbO}_{3}$ by neutron diffraction on powder samples. These compounds contain tetravalent rare earth cations and are all readily prepared at an oxygen pressure of 1 atmosphere. This may be contrasted with the more drastic preparative condi-

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> $\dagger \dagger$ The tolerance factor is defined by:

$$
t=\frac{r_{\mathrm{A}}+r_{\mathrm{o}}}{\sqrt{2\left(r_{\mathrm{B}}+r_{\mathrm{O}}\right)}}
$$

where $r_{\mathrm{A}}, r_{\mathrm{B}}$ and $r_{0}$ are the radii of the $A$ and $B$ metals and oxygen (see for example Goodenough \& Longo, 1970).
tions necessary to form $\mathrm{PrO}_{2}$ (MacChesney, Williams, Sherwood \& Potter, 1964) and $\mathrm{TbO}_{2}$ (MacChesney, Williams, Sherwood \& Potter, 1966) and illustrates the stabilization of high formal oxidation states often observed in the perovskite structure. The magnetic behaviour of these high oxidation state rare earth ions as revealed by neutron diffraction is also of considerable interest.

Recently (Danelon Mastromonaco, Barbariol \& Cocco, 1969), $\mathrm{BaCeO}_{3}$ has been shown to have an orthorhombic cell ( $a=6.214, b=6 \cdot 236, c=8.779 \AA$ ) although earlier workers (Hoffmann, 1935; Smith \& Welch, 1960) reported a simple cubic structure. $\mathrm{BaPrO}_{3}$ and $\mathrm{BaTbO}_{3}$ have previously been indexed as simple cubic ( $a=4 \cdot 354$, Hoffmann, 1935; $a=4 \cdot 285 \AA$, Paletta \& Hoppe, 1966, respectively) although Nàray-Szabò


Fig. 1. The rhombohedral structure of $\mathrm{BaTbO}_{3}$. The relation of the primitive rhombohedral cell to the perovskite cube is shown, and the oxygen rotations about the trigonal axis are indicated for one $\mathrm{BO}_{6}$ octahedron.
(1947) reported a doubling of the unit cell for $\mathrm{BaPrO}_{3}$ ( $a=8 \cdot 708 \AA$ ).

## Preparation and characterization of samples

## Preparation

The starting materials were $\mathrm{BaCO}_{3}, \mathrm{CeO}_{2}$ and $\mathrm{Tb}_{4} \mathrm{O}_{7}$ (all 'Specpure': Johnson Matthey Chemicals Ltd.) and $\mathrm{Pr}_{6} \mathrm{O}_{11}$ ( $99.9 \%$, BDH Chemicals). The $\mathrm{BaCO}_{3}$ was predried by heating in $\mathrm{CO}_{2}$ at $900^{\circ} \mathrm{C}$, the $\mathrm{CeO}_{2}$ was dried in oxygen at $900^{\circ} \mathrm{C}$ and the $\mathrm{Tb}_{4} \mathrm{O}_{7}$ and $\mathrm{Pr}_{6} \mathrm{O}_{11}$ reduced to the sesquioxides by firing in vacuum at $1100^{\circ} \mathrm{C}$. The $\mathrm{BaCO}_{3}$-rare earth oxide mixtures were handled in a glove box filled with dry nitrogen since the oxides readily absorb water and carbon dioxide. The mixtures were fired twice at $1050^{\circ} \mathrm{C}$ for 48 hours in pure oxygen. The compounds thus prepared are white $\left(\mathrm{BaCeO}_{3}\right)$, dark brown $\left(\mathrm{BaPrO}_{3}\right)$ and yellow $\left(\mathrm{BaTbO}_{3}\right)$.

## Analysis

The oxidizing powers of $\mathrm{BaCeO}_{3}$ and $\mathrm{BaTbO}_{3}$ were determined by oxidation of iodide in dilute HCl . The liberated iodine was titrated against standard sodium thiosulphate. The results obtained were $\mathrm{BaCeO}_{3.00 \pm 0.01}$ and $\mathrm{BaTbO} 2.96 \pm 0.01 . \mathrm{BaPrO}_{3}$ is too powerful an oxidizing agent to be easily determined in this way and


Fig. 2 The orthorhombic $\left(\mathrm{GdFeO}_{3}\right)$ structure of $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$. The tilting of the octahedra about the $b$ and $c$ axes may be observed.
the composition $\mathrm{BaPrO}_{2.990 \pm 0.005}$ was found from the weight loss on reaction.

## $X$-rays

X-ray photographs of the three compounds were taken with a Debye-Scherrer camera (diameter 11.46 $\mathrm{cm})$ using copper $K \alpha$ radiation. For $\mathrm{BaTbO}_{3}$ all the lines were indexed on the simple perovskite unit cell, although splittings were visible at high angle ( $d<1 \cdot 0 \AA$ ). These splittings, together with the appearance of extra lines in the neutron diffraction pattern (based on a doubled unit cell with $h k l$ all odd) indicated a rhombohedral distortion.

The X-ray powder photographs of $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$ also showed high-angle splittings and faint extra lines indicative of a doubled unit cell. An orthorhombic distortion was indicated by the presence of reflexions with $h k l$ mixed in the neutron diffraction pattern.

A least-squares analysis of the high angle X-ray reflexions gave the lattice parameters in Table 1. The parameters reported by previous workers agree well with our results (Danelon Mastromonaco et al., 1969; Palleta \& Hoppe, 1966).

Table 1. Lattice parameters

| $\mathrm{BaCeO}_{3}$ | $\mathrm{BaPrO}_{3}$ | $\mathrm{BaTbO}_{3}$ |
| :---: | :---: | :---: |
| $a=6.212 \pm 0.001 \AA$ | $a=6 \cdot 181 \pm 0.001 \AA$ | * $a=8.573 \pm 0.001 \AA$ |
| $b=6.235 \pm 0.001$ | $b=6.214 \pm 0.001$ | $\alpha=90^{\circ} 17^{\prime} \pm 1^{\prime}$ |
| $c=8.781 \pm 0.001$ | $c=8.722 \pm 0.001$ | $\dagger a=6.046 \pm 0.001$ |
| $c / V 2=6.209$ | $c / V 2=6 \cdot 167$ | $\alpha=60^{\circ} 22^{\prime} \pm 1^{\prime}$ |

* Face centered rhombohedral cell. $\dagger$ Primitive rhombohedral cell.

Table 2. Observed and calculated intensities for $\mathrm{BaTbO}_{3}$

| $h k l$ | $I_{\text {cale }}$ | $I_{\text {obs }}$ | $\sigma\left(I_{\text {obs }}\right)$ | $h k l$ | $I_{\text {calc }}$ | $I_{\text {obs }}$ | $\sigma\left(I_{\text {obs }}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 200 | $42 \cdot 7$ | $43 \cdot 3$ | $2 \cdot 1$ | 822 \} | $28 \cdot 7$ | $32 \cdot 2$ | $1 \cdot 6$ |
| 220 | $37 \cdot 7$ | $35 \cdot 6$ | $1 \cdot 6$ | 660 |  |  |  |
| 311 | $14 \cdot 2$ | $14 \cdot 6$ | $1 \cdot 0$ | 751 | $26 \cdot 6$ | $25 \cdot 6$ | $1 \cdot 9$ |
| 222 | $59 \cdot 4$ | $62 \cdot 5$ | $1 \cdot 5$ | 555 |  |  |  |
| 400 | $145 \cdot 4$ | $145 \cdot 6$ | $2 \cdot 1$ | 662 |  |  |  |
| 331 | $7 \cdot 2$ | $6 \cdot 2$ | $1 \cdot 0$ | 840 | $74 \cdot 8$ | $77 \cdot 0$ | $1 \cdot 8$ |
| 420 | $34 \cdot 4$ | $34 \cdot 6$ | $1 \cdot 3$ | 911 |  |  |  |
| 422 | $35 \cdot 0$ | $34 \cdot 5$ | $1 \cdot 1$ | 753 | $34 \cdot 9$ | $37 \cdot 0$ | 1.9 |
| 333 | $4 \cdot 9$ | $4 \cdot 9$ | 0.9 | 842 ) |  |  |  |
| 511 | 4.9 | 4.9 | 0.9 | 664 | $11 \cdot 5$ | 11.0 | $1 \cdot 1$ |
| 440 | 129-1 | $129 \cdot 2$ | $1 \cdot 6$ | 931 | $12 \cdot 5$ | $10 \cdot 2$ | $1 \cdot 5$ |
| 531 |  |  |  | 844 | $57 \cdot 5$ | $56 \cdot 0$ | $1 \cdot 4$ |
| 600 | $43 \cdot 4$ | $42 \cdot 2$ | $1 \cdot 5$ | 933 ) |  |  |  |
| 442 |  |  |  | 771 | $21 \cdot 1$ | $20 \cdot 8$ | $1 \cdot 6$ |
| 620 | $16 \cdot 1$ | $13 \cdot 6$ | $1 \cdot 1$ | 755 |  |  |  |
| 533 | $38 \cdot 4$ | $36 \cdot 2$ | 1.9 | 10,0,0 |  |  |  |
| 622 | 38.4 | 362 | 1. | 860 |  |  |  |
| 444 | $51 \cdot 4$ | $49 \cdot 4$ | $1 \cdot 5$ | 10,2,0 $\}$ | $38 \cdot 0$ | $39 \cdot 7$ | $1 \cdot 7$ |
| 711 | $18 \cdot 2$ | $17 \cdot 2$ | $2 \cdot 0$ | 862 \{ |  |  |  |
| 551 | 182 | 17.2 | 2 | 951 |  |  |  |
| 642 | $31 \cdot 4$ | $30 \cdot 5$ | $1 \cdot 2$ | 753 | $7 \cdot 0$ | $8 \cdot 2$ | $1 \cdot 5$ |
| 731 | $15 \cdot 3$ | $17 \cdot 0$ | $1 \cdot 7$ | 10,2,2 |  |  |  |
| 553 | $15 \cdot 3$ | 17.0 | 1.7 | 666 |  |  |  |
| 800 | $25 \cdot 6$ | $29 \cdot 2$ | $1 \cdot 8$ | 10,4,0 | $17 \cdot 4$ | $16 \cdot 3$ | $2 \cdot 1$ |
| 733 |  |  |  | 864 |  |  |  |
| 820 | $18 \cdot 3$ | $18 \cdot 1$ | $1 \cdot 4$ | 973 |  |  |  |
| 644 |  |  |  |  |  |  |  |

## Neutron diffraction measurements and refinements

Neutron diffraction measurements were made at room temperature on a powder diffractometer at the reactor PLUTO, A.E.R.E., Harwell. The samples were contained in thin-walled vanadium cans. The $\mathrm{BaTbO}_{3}$ measurements were made at a wavelength of $1 \cdot 105 \AA$ from the (311) planes of a copper monochromator with a take-off angle of $60^{\circ}$. The $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$ were examined at a wavelength of $1.497 \AA$ from the (311) planes of a germanium monochromator with a take-off angle of $52^{\circ}$. Absorption and extinction effects were small and no corrections were necessary. The atom positions, scale factor and isotropic temperature factors of $\mathrm{BaTbO}_{3}$ were refined from the observed intensities in space group $R \overline{3} c$ using a least-squares program to refine overlapping reflexions (written by
P. J. Wiseman of this laboratory). The function minimized in this program is $\sum_{i} w_{i}\left(I_{\text {obs }}-I_{\text {calc }}\right)^{2}$ where $w=$ $1 / \sigma^{2}$. All the 29 observed intensities were included in the refinement. The scattering lengths used were: $b_{\mathrm{Tb}}=0.76 \times 10^{-12} \mathrm{~cm} ., \quad b_{\mathrm{O}}=0.577 \times 10^{-12} \mathrm{~cm}$. (The Neutron Diffraction Commission, 1969), and $b_{\text {Ba }}=0.535 \times 10^{-12} \mathrm{~cm}$. The barium value was calculated from the barium: fluorine ratio given by Cooper, Rouse \& Willis (1968), but using a more recent fluorine value of $0.574 \times 10^{-12} \mathrm{~cm}$. (Shull, 1971). The refinement converged after 4 cycles and the final $R$ index (on intensities) was $0 \cdot 037$. The observed and calculated intensities are given in Table 2 and the atom positions in Table 4 (errors are quoted to $\sigma$ ). Refinement of the structure in the non-centrosymmetric space group $R 3 c$ did not improve the $R$ index and atom displace-

Table 3. Observed and calculated intensities for $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$

|  | $\mathrm{BaCeO}_{3}$ |  |  | $\mathrm{BaPrO}_{3}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hkl | $I_{\text {calc }}$ | $I_{\text {obs }}$ | $\sigma\left(I_{\text {obs }}\right)$ | hkl | $I_{\text {calc }}$ | $I_{\text {obs }}$ | $\sigma\left(I_{\text {obs }}\right)$ |
| 200 | $8 \cdot 66$ | $9 \cdot 17$ | $0 \cdot 15$ | 200 | 2.87 | $2 \cdot 67$ | $0 \cdot 16$ |
| 220 | $9 \cdot 86$ | 8.91 | $0 \cdot 14$ | 220 | 3.03 | 2.74 | $0 \cdot 14$ |
| 310 | $3 \cdot 37$ | $3 \cdot 19$ | $0 \cdot 13$ | 310 | $0 \cdot 95$ | 0.87 | 0.17 |
| 311 | $24 \cdot 17$ | 23.85 | $0 \cdot 24$ | 311 | $8 \cdot 36$ | $8 \cdot 42$ | $0 \cdot 16$ |
| 222 | $45 \cdot 44$ | 44.88 | $0 \cdot 33$ | 222 | 18.80 | $18 \cdot 69$ | $0 \cdot 19$ |
| 320 | 1.09 | $0 \cdot 35$ | $0 \cdot 22$ | 321 | $0 \cdot 93$ | $1 \cdot 15$ | $0 \cdot 17$ |
| 321 | 3.82 | $3 \cdot 73$ | $0 \cdot 20$ | 400 | 24.69 | 25.24 | $0 \cdot 21$ |
| 400 | $65 \cdot 67$ | 66.45 | $0 \cdot 28$ | 331 | $3 \cdot 66$ | $3 \cdot 63$ | $0 \cdot 14$ |
| 410 3 | $1 \cdot 41$ | $1 \cdot 19$ | $0 \cdot 10$ | 420 | 3.04 6.68 | 3.45 7.15 | 0.14 |
| 322 3 | 10.17 | 10.05 | 0.10 | 422 | $6 \cdot 68$ | $7 \cdot 15$ | $0 \cdot 16$ |
| 331 | $10 \cdot 17$ | 10.05 | $0 \cdot 14$ | 510 | 0.53 | 0.49 | $0 \cdot 10$ |
| 420 | 8.83 | 9.31 2.24 | $0 \cdot 14$ $0 \cdot 11$ | 431 |  |  |  |
| 421 | 2.34 2 | 23.24 23.57 | 0.11 0.18 | 333 | 2.01 | 1.84 | $0 \cdot 10$ |
| 422 | $22 \cdot 84$ | 23.57 | $0 \cdot 18$ | 511 |  |  |  |
| 430 | $0 \cdot 29$ | $0 \cdot 26$ | $0 \cdot 16$ | 440 | 18.33 | 18.20 | $0 \cdot 22$ |
| 510 | 1.79 | 1.89 | $0 \cdot 11$ | 530 |  |  |  |
| 431 333 | 1.79 5.76 | 5.44 | 0.14 | 531 600 | $13 \cdot 69$ | $13 \cdot 21$ | 0.30 |
| 511 \} | 5.76 | 5.44 | $0 \cdot 14$ | 442 |  |  |  |
| 520 , | 1.09 | $0 \cdot 88$ | $0 \cdot 12$ | 532 | $1 \cdot 35$1.76 | 0.99 | $0 \cdot 14$$0 \cdot 14$ |
| 432 \} | 1.09 | 0.88 | $0 \cdot 12$ | 620 |  | $1 \cdot 61$ |  |
| 521 | $1 \cdot 51$ | $1 \cdot 28$ | $0 \cdot 13$ | 533 |  |  |  |
| 440 | 47.15 | $46 \cdot 68$ | $0 \cdot 30$ | 622 | 11.94 | 11.24 | $0 \cdot 30$ |
| $\left.\begin{array}{l}522 \\ 411\end{array}\right\}$ | 0.74 | $0 \cdot 54$ | $0 \cdot 19$ | 630 542 |  |  |  |
| 530 |  |  |  | 444 | $6 \cdot 20$ | $6 \cdot 30$ | $0 \cdot 18$ |
| 531 | $38 \cdot 54$ | 38.70 | $0 \cdot 49$ | 710 |  | $5 \cdot 42$ |  |
| 600 442 |  |  |  | 543 711 | $5 \cdot 07$ |  | $0 \cdot 24$ |
| 532 | $4 \cdot 43$ | $4 \cdot 50$ | 0.23 | 551 |  |  |  |
| 620 | $5 \cdot 33$ | $4 \cdot 59$ | $0 \cdot 21$ | 720 | $1 \cdot 67$ | 0.77 | $0 \cdot 40$ |
| 621 |  |  |  | 641 |  |  |  |
| 540 | $2 \cdot 28$ | 1.72 | $0 \cdot 37$ | 721 |  |  |  |
| 443 |  |  |  | 642 |  |  |  |
| 541 |  |  |  | 722 544 | $5 \cdot 95$ | $5 \cdot 38$ | $0 \cdot 21$ |
| 533 | $27 \cdot 95$ | $27 \cdot 69$ | $0 \cdot 47$ | 730 | $4 \cdot 22$ | $4 \cdot 11$ | $0 \cdot 24$ |
| 622 |  |  |  | 731 |  |  |  |
| 630 |  |  |  | 533 |  |  |  |
| 542 ) |  |  |  | 800 |  |  |  |
|  |  |  |  | 810 | $3 \cdot 29$ | $2 \cdot 69$ | $0 \cdot 23$ |
|  |  |  |  | 740 |  |  |  |
|  |  |  |  | 733 |  | $2 \cdot 17$ | 0.23 |
|  |  |  |  | 820 | $2 \cdot 55$ |  |  |
|  |  |  |  | 644 |  |  |  |

ments from the special positions of $R \overline{3} c$ were not statistically significant.

The $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$ intensity data were refined similarly in the orthorhombic space group Pbnm. The number of intensities collected ( 25 and 24 respectively) was insufficient to allow all the parameters to be varied and the temperature factors were fixed at the values obtained from the $\mathrm{BaTbO}_{3}$ results. Refinement of the scale factor and the seven positional parameters was continued until the shifts were considerably less than the e.s.d.'s. The final $R$ indices were 0.027 for $\mathrm{BaCeO}_{3}$ and 0.050 for $\mathrm{BaPrO}_{3}$. The observed and calculated intensities are given in Table 3 and the atom positions in Table 4. In Tables 2 and 3 the pseudo-cubic indices are given although the correct rhombohedral and orthorhombic indices were used in the refinement. Scattering lengths, $b_{\mathrm{Ce}}=0.479 \times 10^{-12} \mathrm{~cm}$ (Valentine \& Willis, 1965) and $b_{\mathrm{Pr}_{\mathrm{r}}}=0.44 \times 10^{-12} \mathrm{~cm}$ (Neutron Diffraction Commission, 1969) were used. Attempts to refine all parameters (constraining $B_{\mathrm{O}(1)}=B_{\mathrm{O}(2)}$ : Fig. 2) led to atom positions which were essentially unchanged but gave spurious temperature factors with large standard deviations.

At low temperatures extra reflexions in the neutron diffraction pattern show that $\mathrm{BaTbO}_{3}$ is a $G$-type antiferromagnet with a Néel temperature of $36^{\circ} \mathrm{K}$. A more detailed account of this work, including a determination of the $\mathrm{Tb}^{4+}$ form factor will be published elsewhere. $\mathrm{BaPrO}_{3}$ is not magnetically ordered at $4 \cdot 2^{\circ} \mathrm{K}$.

## Results

From the experimental values of the atom positions some of the important interatomic distances and angles
in the three compounds have been calculated (Table 5 and Figs. 1 and 2). The angles in the $\mathrm{PrO}_{6}$ octahedron are similar to those in $\mathrm{BaCeO}_{3}$ but the standard deviations are much larger $\left(1.8^{\circ}\right)$.

## The rhombohedral perovskite: $\mathrm{BaTbO}_{3}$

In $\mathrm{BaTbO}_{3}$ the $\mathrm{Tb}-\mathrm{O}$ distances are all equal ( $2 \cdot 15 \AA$ ) and the distortion of the octahedron is small. We would expect, therefore, a good correlation between the rotation of the octahedron from the ideal perovskite position about the trigonal axis, and the rhombohedral angle $\alpha$. The angle of rotation $\omega$ may be calculated from the oxygen positions and is related to $\alpha$ as shown by Moreau, Michel, Gerson \& James (1970):

$$
\cos \alpha=\frac{4-\cos ^{2} \omega}{4+2 \cos ^{2} \omega}
$$

The value of $\omega$ calculated is $7^{\circ} 33^{\prime}$, giving $\alpha=60^{\circ} 25^{\prime}$, in excellent agreement with the value of $60^{\circ} 22^{\prime} \pm 1^{\prime}$ derived from the X-ray powder pattern. The magnitude of the rotation is similar to that found in Pb $\left(\mathrm{Zr}_{0.9} \mathrm{Ti}_{0.1}\right) \mathrm{O}_{3}$ where $\omega=8^{\circ}$ (Michel, Moreau, Achenbach, Gerson \& James, 1969; Moreau, et al., 1970). The agreement between the observed and calculated values is rather better in $\mathrm{BaTbO}_{3}$ than in $\mathrm{Pb}\left(\mathrm{Zr}_{0.9} \mathrm{Ti}_{0.1}\right) \mathrm{O}_{3}$ (where the discrepancy is $8^{\prime}$ ), presumably because the latter compound is more distorted, since the metal atoms are displaced along the $\langle 111\rangle$ directions.

The orthorhombic perovskites: $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$
The results for $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$ are in accord with those obtained for other orthorhombic perovskites. The oxygen environment of the large A cation

Table 4. Atom positions in $\mathrm{BaTbO}_{3}, \mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$
( $x, y, z$ are fractions of the unit cell edges, $B$ is in $\AA^{2}$.)
$\mathrm{BaTbO}_{3}(R \overline{3} c)$

| $\mathrm{Ba}^{\text {( }}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $x$ | $y$ | $z$ | $B$ |
| $2 \mathrm{~Tb}^{4+}$ | 2(b) | $0 \cdot 0$ | $0 \cdot 0$ | $0 \cdot 0$ | $0.24 \pm 0.07$ |
| $2 \mathrm{Ba}^{2+}$ | 2(a) | $0 \cdot 25$ | $0 \cdot 25$ | 0.25 | $0.75 \pm 0.10$ |
| $6 \mathrm{O}^{2-}$ | 6(c) | $-0.2876 \pm 0.0006$ | $0.7876 \pm 0.0006$ | 0.25 | $0.89 \pm 0.06$ |
| $R(I)=0.037$ (29 intensities) |  |  |  |  |  |
| $\mathrm{BaCeO}_{3}(\mathrm{Pbnm})$ |  |  |  |  |  |
|  |  | $x$ | $y$ | 2 |  |
| $4 \mathrm{Ba}^{2+}$ | 4(c) | $0.001 \pm 0.001$ | $0.023 \pm 0.002$ | 0.25 |  |
| $4 \mathrm{Ce}^{4+}$ | 4(b) | $0 \cdot 0$ | $0 \cdot 5$ | $0 \cdot 0$ |  |
| $4 \mathrm{O}(1)^{2-}$ | 4(c) | $0.071 \pm 0.004$ | $0 \cdot 487 \pm 0.004$ | $0 \cdot 25$ |  |
| 8O(2) ${ }^{2-}$ | 8(d) | $-0.274 \pm 0.004$ | $0 \cdot 278 \pm 0.003$ | $0.041 \pm 0.002$ |  |
| $R(I)=0.027$ (25 intensities) |  |  |  |  |  |
| $\mathrm{BaPrO}_{3}(\mathrm{Pbnm})$ |  |  |  |  |  |
|  |  | $x$ | $y$ | $z$ |  |
| $4 \mathrm{Ba}^{2+}$ | 4(c) | $0.014 \pm 0.009$ | $0.023 \pm 0.007$ | 0.25 |  |
| $4 \mathrm{Pr}^{4+}$ | 4 (b) | 0.0 | 0.5 | 0.0 |  |
| $4 \mathrm{O}(1)^{2-}$ | 4(c) | $0.071 \pm 0.006$ | $0 \cdot 481 \pm 0.010$ | 0.25 |  |
| 80(2) ${ }^{2-}$ | 8(d) | $-0.271 \pm 0.005$ | $0.271 \pm 0.006$ | $0.037 \pm 0.003$ |  |

is considerably distorted but the $\mathrm{BO}_{6}$ octahedra remain almost regular. The average praseodymium-oxygen distance is slightly smaller than the cerium-oxygen distance $(2 \cdot 22 \AA$ compared with $2 \cdot 24 \AA)$ as expected. The formal coordination number of the A cation is twelve, though Marezio, Remeika \& Dernier (1970) have shown that in the rare earth orthoferrites the lanthanide ion may be effectively only eight coordinate. However, in $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$ twelve coordination is indicated by the absence of sharp discontinuities in the increasing barium-oxygen distances (Table 5). The average $\mathrm{Ba}-\mathrm{O}$ distances increase from $\mathrm{BaTbO}_{3}$ to $\mathrm{BaPrO}_{3}$ and $\mathrm{BaCeO}_{3}$ ( $3 \cdot 03,3 \cdot 11$ and $3 \cdot 13 \AA$, respectively) and are all larger than the value of $3 \cdot 0 \AA$ (Shannon \& Prewitt, 1969) expected for twelve-coordination (cf. discussion below). This may be understood if we consider the barium ion to be located on a site defined by rotations of rigid $\mathrm{BO}_{6}$ octahedra (see below).

Table 5. Interatomic distances ( $\AA$ ) and angles $\left({ }^{\circ}\right)$
Numbers in parentheses give the number of atoms at that distance.

|  | $\mathrm{BaCeO}_{3}$ | $\mathrm{BaPrO}_{3}$ | $\mathrm{BaTbO}_{3}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{M}^{4+}-\mathrm{O}(1)$ | $2 \cdot 24 \pm 0.02$ (2) | $2.23 \pm 0.06$ (2) | $2 \cdot 154 \pm 0.004$ (6) |
| $\mathrm{M}^{4+} \mathrm{O}(2)$ | $2.22 \pm 0.02$ (2) | $2 \cdot 22 \pm 0.03$ (2) |  |
|  | $2.26 \pm 0.02$ (2) | $2 \cdot 22 \pm 0.03$ (2) |  |
| $\mathrm{Ba}-\mathrm{O}(1)$ | $2.67 \pm 0.05$ (1) | $2.58 \pm 0.09$ (1) | $2.807 \pm 0.004$ (3) |
|  | $2 \cdot 93 \pm 0.05$ (1) | $2 \cdot 87 \pm 0.09$ (1) | $3.031 \pm 0.004$ (6) |
|  | $3 \cdot 37 \pm 0.05$ (1) | $3 \cdot 39 \pm 0.09$ (1) | $3 \cdot 264 \pm 0.004$ (3) |
|  | $3 \cdot 60 \pm 0 \cdot 05$ (1) | $3.63 \pm 0.09$ (1) |  |
| $\mathrm{Ba}-\mathrm{O}(2)$ | $2.77 \pm 0.05$ (2) | $2.86 \pm 0.07$ (2) |  |
|  | $2.96 \pm 0.05$ (2) | $2.99 \pm 0.07$ (2) |  |
|  | $3.17 \pm 0.05$ (2) | $3.11 \pm 0.07$ (2) |  |
|  | $3 \cdot 59 \pm 0.05$ (2) | $3.48 \pm 0.07$ (2) |  |
|  | $\mathrm{O}-\mathrm{Tb}-\mathrm{O}$ | $89.44 \pm 0.07$ |  |
|  | $\mathrm{O}(1)-\mathrm{Ce}-\mathrm{O}$ | (2) $89 \cdot 6 \pm 0 \cdot 8$ |  |
|  | $\mathrm{O}(1)-\mathrm{Ce}-\mathrm{O}$ | (2) $88.3 \pm 0.8$ |  |
|  | $\mathrm{O}(2)-\mathrm{Ce}-\mathrm{O}$ | (2) $88 \cdot 6 \pm 0 \cdot 8$ |  |

## Distortions and interatomatic distances in $\boldsymbol{O}$-perovskites

## Lattice parameter relationships

It has not previously been pointed out that relationships between cell parameters and the rotations of the octahedra may be obtained for the orthorhombic as well as for the rhombohedral perovskites. The ideal $\mathrm{GdFeO}_{3}$ structure may be generated from the simple cubic structure by cooperative rotations of regular octahedra about the orthorhombic $\langle 010\rangle$ and $\langle 001\rangle$ directions. If the angles of rotation are $\theta$ and $\varphi$ respectively and $l$ is the B-O distance the orthorhombic lattice parameters become:

$$
a=2 \downarrow 2 l \cos \theta \cos \varphi, b=2 \downarrow 2 l \cos \varphi, c=4 l \cos \varphi
$$

giving (i) $\mathrm{a} \leq c / \sqrt{ } 2<b$ as is generally observed, where $a=c / \sqrt{ } 2$ for zero rotation about $\langle 001\rangle$, and (ii) $l=$ $b c / 4 a$. It is also possible to express the oxygen positional parameters in terms of $l, \theta$ and $\varphi$.

Several series of orthorhombic perovskites (including an extensive set of rare earth oxides $\mathrm{LnBO}_{3}$ ) with different A cations but the same B cation have been prepared. In each series, relation (ii) gives consistent values for the average B-O distances, showing that the small distortions of the $\mathrm{BO}_{6}$ octahedra which do occur, and which vary with the degree of orthorhombic distortion (e.g. $\mathrm{LnFeO}_{3}$ ), in most cases do not significantly effect calculated values of $l$. Also, although atom positions have been determined for only a few $\mathrm{GdFeO}_{3}$-type structures, predicted and experimental values for most of these are in very good agreement: for example $\mathrm{CaTiO}_{3}$, predicted and experimental $1.93 \AA$ (Kay \& Bailey, 1957), and all the orthoferrites from $\mathrm{GdFeO}_{3}$ to $\mathrm{LuFeO}_{3}$, predicted and experimental $2.01 \AA$ (Marezio et al., 1970)

However, when the orthorhombic distortion is small, the calculated values of $l$ appear to be low and this is borne out by a comparison of the calculated and experimental values for $\mathrm{LaFeO}_{3}$ (Marezio \& Dernier, 1971) and $\mathrm{PrFeO}_{3}$ (Marezio et al., 1970) as well as $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}: \mathrm{LaFeO}_{3} ; l(\mathrm{calc}$. $)=1.97 \AA, l($ exptl. $)=2.01 \AA$, $\mathrm{PrFeO}_{3}(1 \cdot 98 \AA, 2 \cdot 01 \AA), \mathrm{BaCeO}_{3}(2 \cdot 20 \AA, 2 \cdot 24 \AA)$ and $\mathrm{BaPrO}_{3}(2 \cdot 19 \AA, 2 \cdot 22 \AA)$. The discrepancy appears to be a consequence of small angular distortions in the oxygen(2) plane which would be expected to increase $a$ and reduce $b$. The $\mathrm{O}(2)-\mathrm{M}-\mathrm{O}(2)$ angles are 88.6 and $91.4^{\circ}$ in $\mathrm{BaCeO}_{3}, 89.1$ and $90.1^{\circ}$ in $\mathrm{BaPrO}_{3}, 88.7$ and $91.3^{\circ}$ in $\mathrm{LaFeO}_{3}$ and 89.3 and $90.7^{\circ}$ in $\mathrm{PrFeO}_{3}$. In contrast, in the orthoferrites mentioned above which are well behaved, the angles are closer to $90^{\circ}$ though it is uncertain whether this is connected with the increased cell distortion. In any case, in both $\mathrm{BaCeO}_{3}$ and $\mathrm{BaPrO}_{3}$ $c / V 2$ is less than $a$ (cf. apparently also $\mathrm{SrZrO}_{3}$ and $\mathrm{SmAlO}_{3}$, Goodenough \& Longo, 1970). For small orthorhombic distortions, therefore, this order of cell parameters is not necessarily associated with Jahn-Teller effects (Goodenough \& Longo). Deviations from $90^{\circ}$ of the $\mathrm{O}(1)-\mathrm{B}-\mathrm{O}(2)$ angles are also observed but appear to have a smaller effect on the cell parameters.

## Variations in metal-oxygen distances

Metal-oxygen distances have not been determined experimentally for the majority of perovskites which are distorted though lattice parameters are often accurately known. Our knowledge of trends of $\mathrm{B}-\mathrm{O}$ or $\mathrm{A}-\mathrm{O}$ distances is thus limited. Relation (ii) however, allows comparisons to be extended to a large number of $O$ perovskites.
In the cubic perovskites a decrease in the $\mathrm{B}-\mathrm{O}$ distance is observed when the size of the A cation is reduced. For example,

$$
\begin{array}{llll}
\mathrm{BaTiO}_{3} & 2.00 \AA & \mathrm{RbUO}_{3} & 2 \cdot 164 \AA \\
\mathrm{SrTiO}_{3} & 1.95 \AA & \mathrm{KUO}_{3} & 2.145 \AA
\end{array}
$$

As the size of the A cation is further decreased, significant distortions occur, and the B-O distances appear to reach limiting values (for example the rare

Table 6. Limiting $B-O$ distances in orthorhombic $\mathrm{ABO}_{3}$ perovskites $(\AA)$
Derived oxygen radii are given in parentheses.

| Sc ${ }^{+}$ | 2.08 (1.35) | parentheses. |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ti ${ }^{+}$ | 2.02 (1.35) | Ti4+ | 1.93 (1.325) | Zr ${ }^{++}$ | 2.06 (1-35) | Hf ${ }^{+}$ | $2 \cdot 055$ (1.34) | Ce ${ }^{4+}$ | $2 \cdot 20$ (1.40) |
| $\mathrm{V}^{3+}$ | 2.01 (1-37) | $\mathrm{V}^{4+}$ | 1.90 (1.33) |  |  |  |  |  |  |
| $\mathrm{Cr}^{3+}$ | 1.98 (1.365) | $\mathrm{Cr}^{4+}$ | 1.88 (1.33) | Mo4+ | 1.99 (1.34) |  |  |  |  |
| $\mathrm{Fe}^{3+}$ | 2.01 (1.365) |  |  | Ru4+ | 1.98 (1-36) |  |  |  |  |
|  |  | $\mathrm{Al}^{3+}$ | 1.90 (1.37) | $\mathrm{Sn}^{4+}$ | 2.02 (1.34) | $\mathrm{Pa}^{5+}$ | $2 \cdot 14$ |  |  |
|  |  | $\mathrm{Ga}^{3+}$ | 1.98 (1.36) | $\mathrm{Pb}^{4+}$ | $2 \cdot 12$ (1-34) | $\mathrm{U}^{5+}$ | $2 \cdot 11$ (1-35) |  |  |
|  |  | In ${ }^{++}$ | $2 \cdot 12$ (1.33) |  |  |  |  |  |  |

earth orthoferrites). In a qualitative sense, this may be viewed as a change from $\mathrm{AO}_{3}$ packing with B cations in octahedral holes to the packing of $\mathrm{BO}_{6}$ octahedra with A cations in twelve coordinate sites.

Some limiting B-O distances for orthorhombic perovskites calculated by relation (ii) are given in Table 6. From these distances oxygen radii (given in parentheses in Table 6) have been derived by subtraction of the 'effective' cationic radii for six-coordination (Shannon \& Prewitt, 1969). The oxygen radii are consistently smaller than that normally associated with sixcoordinate oxygen ( $1 \cdot 40 \AA$, Shannon \& Prewitt). Similarly, metal-oxygen distances for the twelve-coordinate A cation can also be significantly smaller than the sum of the 'effective' ionic radii. For example, in cubic $\mathrm{BaTiO}_{3}$ the experimental value is $2 \cdot 84 \AA$, compared with a calculated sum of $3.00 \AA$, and in $\mathrm{SrTiO}_{3}$ the $\mathrm{Sr}-\mathrm{O}$ distance is $0.08 \AA$ smaller than expected.

Shannon and Prewitt suggest that the short metaloxygen distances observed in the perovskite structure arise from anion-anion repulsions. The deduced oxygen radii in Table 6 are, on the other hand, similar to the 'effective' ionic radius for two-coordinate oxygen ( $1.35 \AA$ ), in line with the fact that the two B cation nearest neighbours are always significantly closer than the four nearest neighbour A cations. However, although the Shannon-Prewitt classification successfully correlates a wide range of metal-oxygen distances by simply taking into account the coordination number of the ions concerned, in view of the inevitable simplicity of the model it is to be expected that some variability in 'effective' ionic radii will occur, even within the same structure. In the present case it seems likely that the short metal-oxygen distances stem from the high lattice energy of the perovskite structure.

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